Reg. No. : ....

Name : ....

K22P 1409

III Semester M.Sc. Degree (CBSS – Reg./Sup./Imp.) Examination, October 2022
(2019 Admission Onwards)
MATHEMATICS
MAT3C12 – Functional Analysis

Time : 3 Hours Max. Marks : 80

PART – A

Answer **any four** questions from Part **A**. **Each** question carries **4** marks.

1. State and prove Jensen's inequality.

2. Prove that a normed space X is a Banach space if and only if every absolutely summable series of elements in X is summable in X.

3. Let X be a Banach space, Y be a normed space and (F_n) be a sequence in BL(X, Y) such that sequence $(F_n(x))$ converges in Y for every $x \in X$. For every $x \in X$, define $F(x) = \lim_{n \to \infty} F_n(x)$. If E is totally bounded subset of X, prove that $(F_n(x))$ converges uniformly for $x \in E$.

4. State and prove bounded inverse theorem.

5. State and prove Schwarz inequality.

6. Among the all l^p spaces, $1 \le p \le \infty$, prove that only l^2 is an inner product space.

PART – B

Answer **any four** questions from Part **B** without omitting any unit. **Each** question carries **16** marks.

Unit – I

7. a) State and prove Riesz Lemma.

   b) Let X be a normed space and Y be a finite dimensional subspace of X. Prove that Y is closed in X.

P.T.O.

8. a) Let $\|\ \|$ and $\|\ \|'$ be norms on a linear space X. Prove that $\|\ \|$ is equivalent to $\|\ \|'$ if and only if there are $\alpha > 0$ and $\beta > 0$ such that $\beta\|x\| \leq \|x\|' \leq \alpha\|x\|$ for all $x \in X$.

   b) Prove that the norms $\|\ \|_1$, $\|\ \|_2$ and $\|\ \|_\infty$ on $\mathbb{K}^n$ are equivalent.

9. a) State and prove Hahn Banach extension theorem.

   b) Prove that a banach space cannot have denumerable basis.

Unit – II

10. State and prove uniform boundedness principle.

11. a) State and prove open mapping theorem.

    b) Give an example to show that the open mapping theorem may not hold normed spaces.

12. Prove that the coefficient functionals corresponding to a Schauder basis for a Banach space X are continuous.

Unit – III

13. a) Let $\{u_\alpha\}$ be an orthonormal set in a Hilbert space H. Prove that $\{u_\alpha\}$ is an orthonormal basis for H if and only if $x \in H$ and $\langle x, u_\alpha \rangle = 0$ for all α implies $x =$

    b) Let $H = l^2$ and for $n = 1, 2, \ldots$ $u_n = (0, \ldots, 0, 1, 0, 0, \ldots)$ where 1 occurs only in the n^{th} entry. Prove that $\{u_n : n = 1, 2, \ldots\}$ is an orthonormal basis for H.

    c) Let F be a subspace of an inner product space X and $x \in X$. Prove that $y \in F$ is a best approximation from F to x if and only if $x - y \perp F$ and in that case $\text{dist}(x, F) = \langle x, x - y \rangle^{\frac{1}{2}}$.

14. State and prove Riesz representation theorem.

15. a) State and prove Bessel's inequality.

    b) Let E be a nonempty closed convex subset of a hilbert space H. Prove that there exist a unique best approximation from E to x for each $x \in H$.

    c) Ler $f \in H'$ and $y \in H$ be the representer of f. Prove that $\|f\| = \|y\|$.
